HIGH GAIN FEL AMPLIFICATION OF CHARGE MODULATION CAUSED BY A HADRON *

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Abstract

In the coherent electron cooling (CeC) scheme [1,2], a modulation of the electron beam density induced by a co-propagation hadron is amplified in a high gain FEL. The resulting amplified modulation of electron beam, its shape, form and its lethargy determine several important properties of the coherent electron cooling. In this paper we present both analytical and numerical evaluations of the corresponding Green functions (using codes RON [3] and Genesis [4]). We also discuss the influence of the electron beam parameters on the FEL response.

INTRODUCTION

The main advantage of coherent electron cooling is that it promises a very short cooling time – under an hour - for high-energy hadron colliders such as RHIC and LHC [1,2]. Strong cooling, in return, has the potential of significant luminosity increases in hadron and electron-hadron colliders [2].

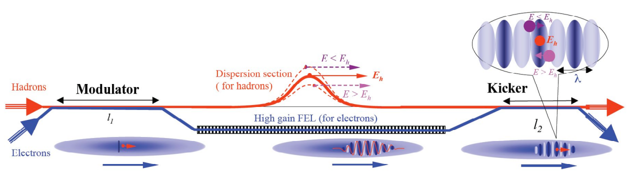

Figure 1: A scheme of a coherent electron cooling

The CeC scheme shown in Fig.1 comprises three parts: The Modulator, the FEL Amplifier for electron and Longitudinal Dispersion for Hadrons, and the Kicker. In CeC electrons and hadrons should have the same relativistic factor: $\gamma_o = E_e / m_e c^2 = E_h / m_h c^2$. In essence, the CeC principles of operation are as follows (see [1,2] for more details): In the modulator, individual hadrons attract electrons and create centers of local density modulation at or near the position of individual hadrons – see [5] for the description of the process. After half of an e-beam plasma oscillation, the electron beam density perturbation attains a total excess charge of –2Ze. In the FEL-amplifier – the main subject of this paper - this electron-beam charge-density modulation is amplified with exponential FEL growth. The spike of electron density modulation evolves in the FEL and becomes a wave-packet of modulation with a period of the FEL wavelength $\lambda_o = \lambda_w (1 + a_w^2)/2\gamma_o^2$, (where λ_w and, a_w respectively, are the wiggler period and wiggler parameter). Most importantly, the modulation contains G_{FEL} -times larger charge. At the same time hadrons traverse the *dispersion section* which correlates their energy and arrival time at the kicker relative to the modulator: $(t - t_o)\mathrm{v_o} = \text{-}\mathrm{D} \cdot \delta$. Fine tuning the CeC provides for synchronization between the space-charge wave-packet induced by a hadron in such a way that the hadron with central energy, E_o, arrives at *the kicker section* just on the top of the pancake of increased electron density (induced by the same hadron in the kicker), wherein the longitudinal electric field is zero. Hadrons with higher energy will arrive at the kicker ahead of their respective pancake in the electron beam, and will be pulled back (decelerated) by the coherent field of the electron beam; we note that positively charged hadrons are attracted to high-density pancakes of electrons. Similarly, a hadron with lower energy falls behind and, as a result will be dragged forward (accelerated) by the clump of electron density. This interaction reduces longitudinal phase space volume of the hadron beam, i.e. effectively cools it.

A decrement of transverse cooling, if desired, can be introduced by the introduction of coupling between the longitudinal and transverse degrees of freedom [1], and it can be as powerful as the longitudinal one.

Comprehensive studies of coherent electron cooling were initiated about ten months ago. These include both theoretical and computational analysis of processes in the modulator, the FEL, the dispersion section and the kicker. Progress of these studies is described elsewhere [5]. The most important conclusion for this paper is that there are clear theoretical [6] and numerical [7] ways of determining the exact 3D time-dependent response of uniform electron plasma to the presence of an ion, moving in both longitudinal and transverse direction (in the co-moving reference frame of the electron beam). Thus, the distribution of the electron beam at the entrance of the FEL, as modified by its interaction with the hadron(s) in the modulator, can be calculated in detail.

In this short paper we focus on the amplification of this modified distribution, i.e. the imprint of the hadrons in the electron beam - in a high-gain FEL. Furthermore, we will focus on the longitudinal part of the FEL Green-function (see below)

FEL'S GREEN FUNCTION

Even though evolution of the optical power in high-gain, single-pass FEL is well studied and well-described in a number of publications, the time-dependent FEL response on a δ-function type disturbance and especially

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the evolution of the density modulation through the FEL has not been thoroughly studied. In general terms, our goal –as shown in Fig. 2 – is to find a kernel of an integral transformation (a linear operator) of the electron beam distribution while propagating in an FEL:

$$f\left(\vec{r}_\perp,\vec{p},t;z\right)=$$
$$\int G\left(\vec{r}_\perp,\vec{r}_\perp{}',\vec{p},\vec{p}',t,t';z\right)f\left(\vec{r}_\perp{}',\vec{p}',t';0\right)d\vec{r}_\perp{}'d\vec{p}'dt' \quad (1),$$

where $G\left(\vec{r}_\perp,\vec{r}_\perp{}',\vec{p},\vec{p}',t,t';z\right)$ is nothing else but a Green function corresponding to a δ-function distortion of the electron-beam distribution at the entrance to the FEL (z=0):

$$\delta f\left(\vec{r}_\perp,\vec{p},t;0\right)=\delta\left(\vec{r}_\perp{}'-\vec{r}_\perp\right)\delta\left(\vec{p}'-\vec{p}\right)\delta\left(t'-t\right). \quad (2)$$

We are considering an FEL with time independent parameters, and therefore the Green function does not depend on time explicitly, i.e. it is a function of the time difference between events:

$$G\left(....,t,t';z\right)=G\left(...,\tau=t-t';z\right) \quad (3)$$

In addition, the Green function satisfies causality conditions, i.e. it is equal to zero outside the light cone:

$$G\left(\vec{r}_\perp,\vec{r}_\perp{}',\vec{p},\vec{p}',t,t';z\right)\equiv 0 \;\; \forall \;\; \left|\vec{r}-r'\right|>c\left|t-t'\right| \quad (4)$$

with the main practical implication for an FEL that the response is non-zero only within a slippage distance (time) in the FEL from the location of initial perturbation.

Naturally, the response also strongly depends on the parameter of the FEL wiggler, associated focusing, wiggler errors, etc. Finding an analytical expression for the Green function of an arbitrary FEL is non-realistic and it is a natural problem for time-resolved 3D FEL codes.

Nevertheless, in the interest of deeper understanding of the process, we wish to simplify it to a solvable problem, which allows a comparison with the 1D FEL solution.

One simplification can be done for a case when the electron beam has only a modification to its density distribution[1], but not in velocity and energy, i.e.

$$\delta f\left(\vec{r}_\perp,\vec{p},t;0\right)=\delta\left(\vec{r}_\perp{}'-\vec{r}_\perp\right)\delta\left(t'-t\right)$$

Diffraction of the optical radiation in the FEL weakens the dependence on the transverse position of the initial distortion. Furthermore, the most interesting information for the kicker is the longitudinal density modulation [1], which is integral over both the momenta and transverse coordinates.

This is the reason why we decided, as the first step in these studies, to focus on a 1D Green-function for the longitudinal density modulation:

$$\rho(t;z)=\int G(\tau;z)\rho(t-\tau;z)d\tau;$$
$$\delta\rho(t;0)=\delta(t), \quad (5)$$

[1] Such situation can occur after a half of plasma oscillation in electron beam when density distortion is at its maximum.

where $\rho(t;z)=\int f\left(\vec{r}_\perp,\vec{p},t;z\right)d\vec{r}_\perp{}'d\vec{p}'$. This approach allows us to compare computer simulations with 1D FEL theory and to check the validity of concept used in CeC.

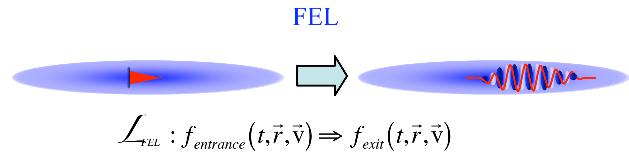

Figure 2: The process of interest for this paper – how a perturbation induced in electron beam distribution in the modulator by the hadron is amplified by a high-gain FEL?

GREEN FUNCTION OF 1D FEL

The linear theory of the 1D FEL is well developed and will use here a number of results from [8], even though we will use more traditional notations.

General qualitative features of the FEL response are also well known – the Green function is a wave-packet, a rather smooth envelope modulated with the FEL frequency:

$$G(\tau;z)=\mathrm{Re}\left(\tilde{G}_z(\tau)e^{i\omega_o\tau}\right)\omega_0=\frac{2\pi c}{\lambda_o}; \quad (6)$$

The total extent of the envelope is equal to the total slippage in the FEL wiggler: $\Delta=N_w\lambda_o$, while the peak of the envelope is located at about one third of the slippage length from the origin. The later is the consequence of the fact that group velocity (of a wave-packet) is equal to one third of the speed of light plus two thirds of the average longitudinal velocity of the electron in the wiggler, v_z [8]:

$$\mathrm{v}_g=\frac{c+2\left\langle \mathrm{v}_z\right\rangle}{3}=c\left(1-\frac{1+a_w^2}{3\gamma_o^2}\right). \quad (7)$$

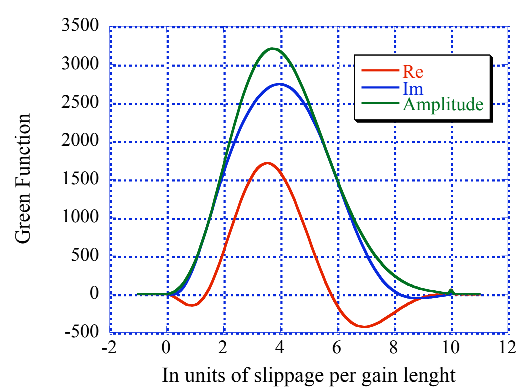

Figure 3: The amplitude, the real and the imaginary part of the Green function envelope after propagating ten gain-lengths in the FEL. For comparison with 3D FEL in the following section – the gain length is 2 m, or 40 periods for amplitude. Thus, the unit slippage in this figure corresponds to 40 optical wavelengths.

The duration of such a wave-packet (i.e., the thickness of the individual pancake stack) is equal to the coherence length of SASE FEL radiation [8,9]. In 1D theory [8] a large number of analytical solutions do exist in the Fourier domain for cases including the effects of space charge and energy spread (we used a Lorentzian distribution). Apart from a poor convergence and a need for careful error analysis, these tools are sufficient for calculating the Green-function of a 1D FEL. In 1D theory everything is naturally scaled by the gain length [8] and Fig. 3 shows the Green function in a ten-gain-lengths FEL as function of the slippage in these units.

Detailed studies of the green function show that its maximum is located at 3.744 slippage units, i.e. just a bit further than the expected 3 1/3 slippage units. The Green function (which oscillates) had effective RMS length [1] of 1.48 slippage units.

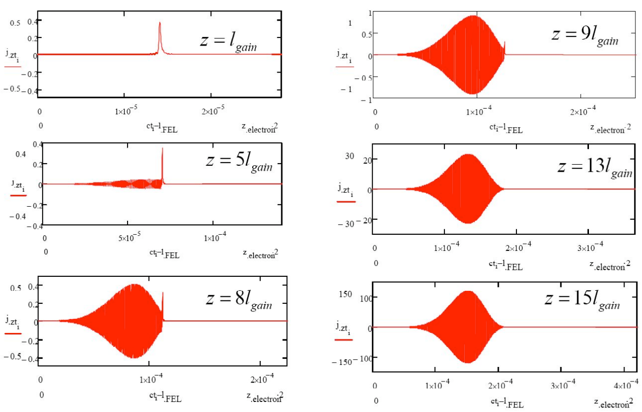

Figure 4: Evolution of the initial distortion of the electron beam density caused by a proton in the CeC modulator in FEL using 1D FEL approach for eRHIC CeC (see table below). The figures show the location along z measured in FEL-power gain-lengths. Note that the horizontal axis is in meters and direction is reversed – the origin is located on the right side of the wave-packet.

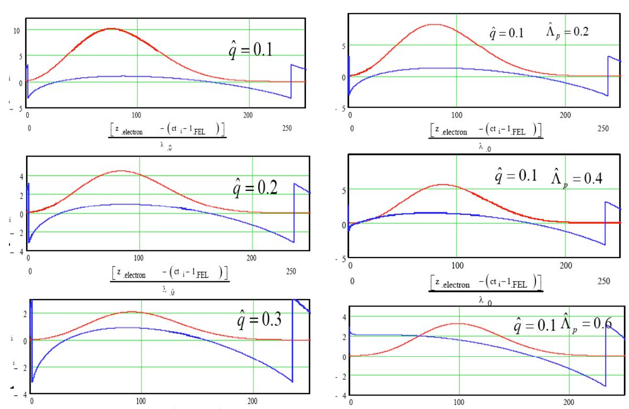

Figure 5: Dependence of the Green-function envelope's amplitude (red line) and its phase (blue line) on energy spread $\hat{q}$ parameter and on space charge $\hat{\Lambda}_p$ parameter (see definitions in [8]). The horizontal axis is in units of optical wavelengths. Total slippage is 240 wavelengths.

Similar features are observed in analytical evaluation of the evolution (in 1D FEL) of the initial distortion of electron beam density caused by a proton (see Fig. 4). After the process of initial formation of the wave-packet, it scales accordingly in expected way. Inclusion of the energy spread, see Fig.5, into the equation does not change the overall qualitative picture, but affect the FEL gain through its gain-length and a slight shift of the peak of the envelope.

Addition of the space charge also affects the gain length and can cause a shift of the envelope and, naturally, its phase. The group velocity can increase by 20-25% for a space charge dominated case ($\hat{\Lambda}_p \sim 1$).

GREEN FUNCTION OF 3D FEL

As the first test, we selected parameters of an FEL amplifier we considered to be suitable to cool 250 GeV protons in RHIC [2]. These parameters give an amplitude gain-length of 2 meters. They had not been optimized and the length of the FEL wiggler was used as parameter. Table 1 lists the main parameters of the FEL system.

Table 1: Main FEL parameters

Energy, MeV	136.2	γ	266.45
Peak current, A	100	λ_o, nm	700
Bunchlength, psec	50	λ_w, cm	5
Emittance, norm	5 mm mrad	a_w	0.994
Energy spread	0.03%	Wiggler	Helical

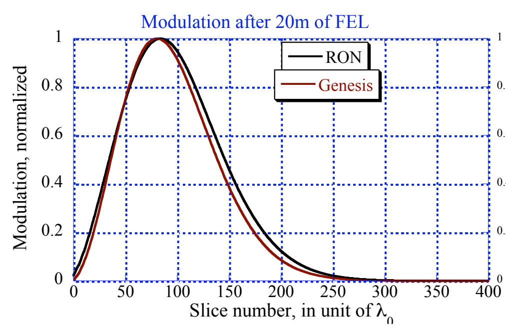

Figure 6: Comparison of the normalized envelopes the Green functions, i.e. the electron beam density modulation caused by a short, local density at its entrance, calculated by the RON and GENESIS FEL codes. The data is evaluated after a 20 m long wiggler.

We used 3D FEL codes, RON [3] and Genesis 1.3 [4], which are based on completely different approaches. RON used multiple frequencies and FFT to simulate the time response to a short spike of the modulation in the electron beam. The Genesis 1.3 was ran in the time-domain mode with only one of 1000 slices (a slice had length of λ_o) had been modulated with 1e-4 bunching value, while the rest of slides have quiet loading. Results of both 3D simulations compare very well with each other (see Fig.6).

Evolution of the wave-packets of the bunch modulation and optical power, shown in Figs. 8 and 9, can be described by their peak values (maxima) and by the location of the maxima. After the usual period of establishment of the exponential regime both the modulation and optical power grow with the correct gain-length (40 period or 2 m for amplitude and 20 periods or 1 m for the power).

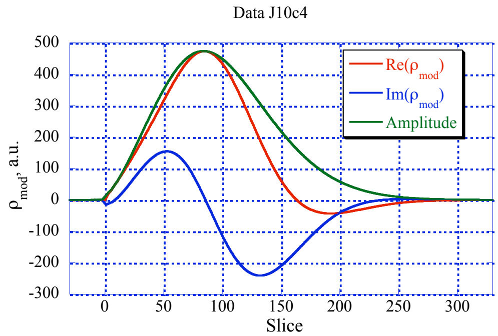

Figure 7: The Green-function envelope calculated by RON at z=20m, i.e. after 10 gain-length FEL, resembles all the features seen in the 1D case (Fig.3).

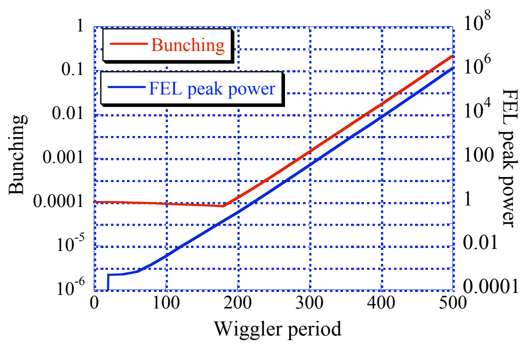

Figure 8: Evolution of the maximum bunching in the e-beam and the FEL power simulated by Genesis.

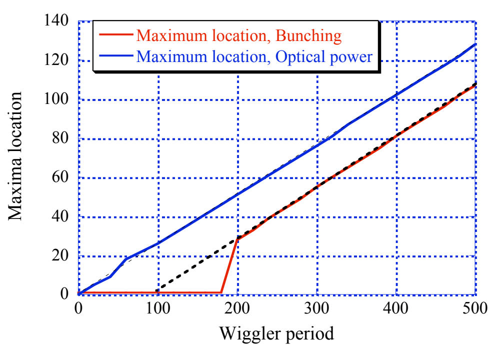

Figure 9: Evolution of the maxima locations in the e-beam bunching and the FEL power simulated by Genesis.

Nevertheless, one should notice that it takes four and a half gain-lengths for bunching to grow above the initial 10^{-4} level. The location of the maxima, both for the optical power and the bunching progresses with a lower speed compared with prediction by 1D theory. It corresponds to a lower group velocity compared with eq. (7):

$$v_g \cong \frac{c + 3\langle v_z \rangle}{4} = c\left(1 - \frac{3}{8}\frac{1 + a_w^2}{\gamma_o^2}\right),$$

i.e. electron beam plays 75% role in its value. It is also noticeable that the electron beam modulation lags behind the optical power – and fig. 10 illustrates the initial stages of this development - and effectively “misses” about two and a half gain-length of the propagation from the origin. The Green function after 10 gain-lengths (which oscillates) had also smaller effective RMS length [1] of 0.96 slippage units (i.e. about 38 optical wavelengths, or 27 microns).

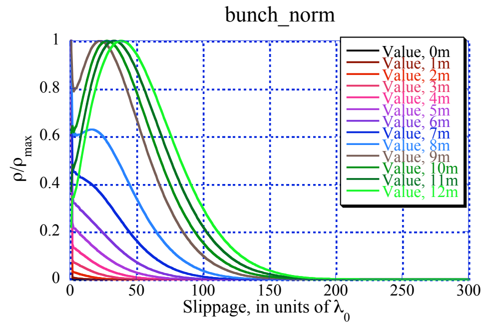

Figure 10. Evolution of the bunching envelope.

CONCLUSIONS

These initial detailed studies did not result in any significant deviation from our initial CeC estimations [1,2]. At the same time, we found a number of new and interesting details to pursue further.

REFERENCES

[1] V.N. Litvinenko, Y.S. Derbenev, Proc. of FEL’07 Conference, Novosibirsk, Russia, August 27-31, 2007, p.268, http://accelconf.web.cern.ch/accelconf/f07

[2] V.N. Litvinenko, Y.S. Derbenev, EPAC’08 Proc., 2008, p.2548, http://accelconf.web.cern.ch/AccelConf/e08

[3] R.J. Dejus, O.A. Shevchenko, N.A. Vinokurov, Nuclear Instruments and Methods in Physics Research, **A 445**, 2000, 19

[4] S. Reiche, DESY preprint, DESY-THESIS-2000-012, 2000, http://pbpl.physics.ucla.edu/~reiche

[5] PROGRESS WITH FEL-BASED COHERENT ELECTRON COOLING. V.N. Litvinenko et al., these proceedings

[6] The Dynamics of Ion Shielding in an Anisotropic Electron Plasma, G. Wang and M. Blaskiewicz, Accepter for publication in Physics Review E.

[7] VORPAL Simulations Relevant to Coherent Electron Cooling, G.I. Bell et al, EPAC’08 Proc., 2008, THPC085, http://accelconf.web.cern.ch/AccelConf/e08

[8] E.L.Saldin, E.A.Schneidmiller, M.V.Yurkov, The Physics of Free Electron Lasers, Springer, 1999

[9] C.Pelegrinni, NIM **A475**, 177, (2001) p.1